

# A method for evaluation of uncertainties of noise parameter measurement

Marek Schmidt-Szałowski and Wojciech Wiatr

**Abstract** — The assessment of uncertainties of a two-port noise parameters measurement, presented in the paper, relies on modeling of sources of errors and an investigation of propagation of the errors through a measurement system. This approach is based on a simplified additive error model and small-change sensitivity analysis. The evaluated uncertainties agrees with those observed in experiments. This method may be implemented in automatic noise measurement systems for on-line uncertainty assessment and for optimization of the design of an experiment.

**Keywords** — microwaves, noise metrology, measurement system, four noise parameters, sensitivity analysis, error propagation, uncertainty.

## 1. Introduction

The trade-off between the measurement through-put and the measurement accuracy is a well-known dilemma encountered in the practice of industrial measurements. This problem becomes critical in case of very sensitive instrumentation like radiometers and noise figure meters, whose accuracy can be essentially improved by extending measurement time. In general, two complementary approaches can be useful in such cases: uncertainty analysis, which enables to predict the accuracy of a measurement, and design of experiment, which tells how to conduct a measurement to obtain the highest accuracy. Although both approaches are well established and commonly used in many fields of engineering they are barely exploited in noise parameter measurements [1–5]. As a result, even state-of-the-art noise-measurement systems does not provide on-line evaluation of accuracy.

The uncertainty of the two-port noise parameters can be estimated either numerically or analytically. In the first approach, called also perturbation method, the two-port parameters are repeatedly calculated from artificially randomly perturbed values of the system parameters and the observed values. This approach allows for various probability distributions of errors and accounts for non-linearities of the model but is very time-consuming. The latter approach based on small-change sensitivities relies on the investigation of the error propagation through the system. This approach is much faster and sufficiently accurate for typical noise measurement systems [6].

This paper concerns the analytical method of an uncertainty analysis. First, a method for determining of the noise pa-

rameters of a microwave receiver is described. Secondly, measurement errors are analyzed and a simple error model is introduced. Then, the error propagation is analyzed and the confidence intervals are calculated for all noise parameters. Finally, the results of total-power radiometer calibrations are presented as exemplary applications of this approach.

## 2. The model of the measurement system

In this paper an identification of the noise parameters is discussed for the case of a microwave receiver calibration. The receiver is characterized in terms the noise correlation matrix  $\mathbf{C}_r$ , and the small-signal parameters: the input reflection coefficient  $\Gamma_r$  and the power gain  $g_r$ . A multi-state noise generator attached to the input of the receiver consist of a bi-state noise source and a multi-state mechanical tuner. The measurement set-up shown in Fig. 1 allows one to observe the output noise temperature  $T_r$  as a function of generator parameters: the reflection coefficient  $\Gamma_g$  and the noise temperature  $T_g$ .

According to [7] the receiver parameters may be determined from a seven-term model

$$\boldsymbol{\beta}^T \mathbf{A} \boldsymbol{\beta} + \mathbf{a}^T \boldsymbol{\beta} + a = 0, \quad (1)$$

where seven-element real vector  $\boldsymbol{\beta}$  contains unknown receiver parameters, while matrix  $\mathbf{A}$ , vector  $\mathbf{a}$ , and coefficient  $a$  depend on the generator state

$$\boldsymbol{\beta} = \begin{bmatrix} \Re \Gamma_r \\ \Im \Gamma_r \\ g_r \\ g_r c_{r11} \\ g_r c_{r22} \\ g_r \Re c_{r21} \\ g_r \Im c_{r21} \end{bmatrix}, \quad \mathbf{A} = T_r \begin{bmatrix} |\Gamma_g|^2 & 0 & \dots & 0 \\ 0 & |\Gamma_g|^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix},$$

$$\mathbf{a} = \begin{bmatrix} -2T_r \Re \Gamma_g \\ 2T_r \Im \Gamma_g \\ -T_g(1 - |\Gamma_g|^2) \\ -|\Gamma_g|^2 \\ -1 \\ 2\Re \Gamma_g \\ 2\Im \Gamma_g \end{bmatrix}, \quad a = T_r. \quad (2)$$

Equation (1) is real, quadratic with respect to  $\beta_1$  and  $\beta_2$  but linear with respect to the other unknowns. It is worth noting that a separate measurement of  $\Gamma_r$  is not required since its value is to be determined simultaneously with the other receiver parameters [7].

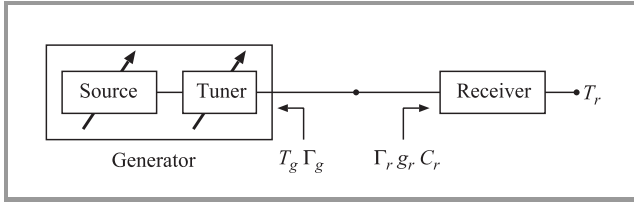


Fig. 1. The measurement set-up.

The deterministic model (1) does not fit reality because  $\mathbf{A}$ ,  $\mathbf{a}$ , and  $a$  are affected by various errors. Typically for small and moderate errors an additive error model may be used

$$\boldsymbol{\beta}^T \mathbf{A} \boldsymbol{\beta} + 2\mathbf{a}^T \boldsymbol{\beta} + a = \varepsilon, \quad (3)$$

where  $\varepsilon$  is a random variable representing errors of  $\Gamma_g$ ,  $T_g$ , and  $T_r$ . It should be emphasised that error  $\varepsilon$  does not account for all types of errors emerging in the system but only those that produce random dispersion of  $\boldsymbol{\beta}$ , i.e. related to type-A uncertainty [8]. Usually, it is assumed that  $\varepsilon$  is distributed according to a normal distribution with null mean and variance  $\sigma^2$ . Evaluation of  $\sigma$  is the most difficult part of the the uncertainty analysis. To simplify this task, it is assumed that error  $\varepsilon$  results from two independent sources:

- Errors related to tuner calibration errors and tuner nonrepeatability, which are modeled as uncorrelated random errors of  $\Gamma_g$  with constant variance  $\text{var } \Gamma_g$  and random errors of  $T_g$  with variance  $\text{var } T_g$  proportional to  $T_{g\text{exc}}^2$ , where  $T_{g\text{exc}} = T_g - T_0$ .
- Errors caused by finite time of integration of noise power  $T_r$ , which are modeled as random errors of  $T_r$  with variance  $\text{var } T_r$  proportional to  $T_r^2$ .

The values of  $\text{var } \Gamma_g$ ,  $\text{var } T_g$ , and  $\text{var } T_r$  can be estimated for each frequency of interest on the basis of long term monitoring and general knowledge of the measurement system.

Owing to the assumed independence of errors the variance  $\sigma^2$  can be expressed in an additive form

$$\sigma^2 = \left| \frac{\partial \varepsilon}{\partial \Gamma_g} \right|^2 \text{var } \Gamma_g + \left( \frac{\partial \varepsilon}{\partial T_g} \right)^2 \text{var } T_g + \left( \frac{\partial \varepsilon}{\partial T_r} \right)^2 \text{var } T_r, \quad (4)$$

where sensitivities  $\partial \varepsilon / \partial \cdot$  are calculated from Eqs. (1) and (2)

$$\frac{\partial \varepsilon}{\partial \Gamma_g} = 2 \left( c_{r21} + \Gamma_g (g_r T_g - c_{r11}) - T_r \Gamma_r^* \zeta \right),$$

$$\frac{\partial \varepsilon}{\partial T_g} = -g_r (1 - |\Gamma_g|^2),$$

$$\frac{\partial \varepsilon}{\partial T_r} = |\zeta|^2, \quad \zeta = 1 - \Gamma_r \Gamma_g.$$

### 3. Identification method

The value of  $\boldsymbol{\beta}$  can be determined on the basis of a series of measurements of  $T_r$  made for several states of the generator through solving a system of equations

$$\boldsymbol{\beta}^T \mathbf{A} \boldsymbol{\beta} + \mathbf{a}^T \boldsymbol{\beta} + a = \varepsilon \quad \text{for } i = 1, \dots, M. \quad (5)$$

Since these equations are nonlinear (quadratic) the deterministic case  $M = 7$  usually leads to more than one solution  $\boldsymbol{\beta}$  [7] and additional knowledge is needed to choosing the proper one. For this reason the system (5) should be overdetermined which also benefits in better immunity to random errors.

According to the Gauss-Markov theorem a generalized least squares estimator  $\mathbf{b}$  is the best linear unbiased estimator of  $\boldsymbol{\beta}$  defined as in model (3) [9]. If errors  $\varepsilon$ , related to different states of the generator, are statistically independent  $\mathbf{b}$  can be obtained by minimizing the sum of squared residuals

$$\phi(\boldsymbol{\beta}) = \sum_{i=1}^M \sigma_i^{-2} e_i^2(\boldsymbol{\beta})$$

$$\text{with } e_i(\boldsymbol{\beta}) = \boldsymbol{\beta}^T \mathbf{A}_i \boldsymbol{\beta} + \mathbf{a}_i^T \boldsymbol{\beta} + a_i. \quad (6)$$

Since the normal equation  $\partial / \partial \boldsymbol{\beta} \phi(\boldsymbol{\beta}) = 0$  is nonlinear (as the model (3) is) and it can be solved using the Gauss-Newton method. Once the estimator  $\mathbf{b}$  is known the covariance matrix  $\text{cov } \mathbf{b}$  can be calculated [9]

$$\text{cov } \mathbf{b} = s^2 \mathbf{M}^{-1} \quad \text{with } \mathbf{M} = \sum_{i=1}^M \sigma_i^{-2} \frac{\partial e_i(\mathbf{b})}{\partial \boldsymbol{\beta}} \left( \frac{\partial e_i(\mathbf{b})}{\partial \boldsymbol{\beta}} \right)^T, \\ s^2 = \frac{1}{M-7} \sum_{i=1}^M \sigma_i^{-2} e_i^2(\mathbf{b}). \quad (7)$$

Factor  $s^2$  can be treated as a measure of goodness of the error model and should equals 1 for the the best model. The receiver noise parameters  $T_{\min}$ ,  $T_N$ , and  $\Gamma_{\text{opt}}$  can be calculated from  $\boldsymbol{\beta}$ . To properly account for their correlations an auxiliary vector  $\mathbf{t}$  may be introduced

$$\mathbf{t} = \begin{bmatrix} T_{\min} \\ T_N \\ \Re \Gamma_{\text{opt}} \\ \Im \Gamma_{\text{opt}} \end{bmatrix} = \frac{1}{2\beta_3} \begin{bmatrix} \beta_5 - \beta_4 + \beta_3 T_N \\ \sqrt{(\beta_4 + \beta_5)^2 - 4\beta_6^2 - 4\beta_7^2} \\ \vartheta \beta_3 \beta_6 \\ \vartheta \beta_3 \beta_7 \end{bmatrix} \\ \text{with } \vartheta = \frac{2}{\beta_4 + \beta_5 + \beta_3 t_2}. \quad (8)$$

Hence

$$\text{cov} \mathbf{t} = \partial \mathbf{t} / \partial \boldsymbol{\beta} \text{ cov } \boldsymbol{\beta} (\partial \mathbf{t} / \partial \boldsymbol{\beta})^T, \quad (9)$$

where

$$\begin{aligned} \frac{\partial \mathbf{t}}{\partial \boldsymbol{\beta}} &= \frac{1}{\beta_3} \times \\ &\times \begin{bmatrix} 00 - t_1 & \frac{\nu-1}{2} & \frac{\nu+1}{2} & -2\frac{\beta_6}{t_2} & -2\frac{\beta_7}{t_2} \\ 00 - t_2 & \nu & \nu & -4\frac{\beta_6}{t_2} & -4\frac{\beta_7}{t_2} \\ 00 - t_3 & -\vartheta t_3 \frac{\nu+1}{2} & -\vartheta t_3 \frac{\nu+1}{2} & 2\frac{t_3^2}{t_2} + \vartheta & 2\frac{t_3 t_4}{t_2} \\ 00 - t_4 & -\vartheta t_4 \frac{\nu+1}{2} & -\vartheta t_4 \frac{\nu+1}{2} & 2\frac{t_4^2}{t_2} & 2\frac{t_4 t_3}{t_2} + \vartheta \end{bmatrix} \\ &\text{with } \nu = \frac{\beta_4 + \beta_5}{t_2}. \end{aligned} \quad (10)$$

Finally the uncertainties of the noise parameters can be determined as

$$\begin{aligned} \text{unc } T_{\min} &= 2\sqrt{\text{var } T_{\min}}, \quad \text{unc } T_N = 2\sqrt{\text{var } T_N}, \\ \text{unc } \Gamma_{\text{opt}} &= 2\sqrt{\text{var } \Re \Gamma_{\text{opt}} + \text{var } \Im \Gamma_{\text{opt}}}. \end{aligned} \quad (11)$$

The correlation coefficient, needed in some applications, can be also determined from  $\text{cov } \mathbf{b}$ .

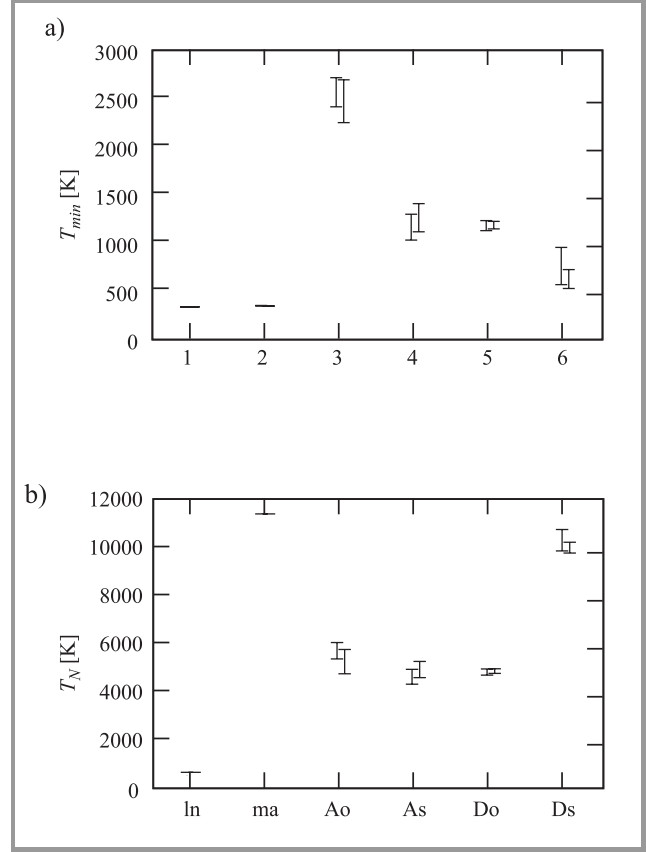
## 4. Experimental results

The described method for identification of the noise parameters was used to calibrate a total-power radiometer (TPR) with various two-ports attached to its input. In the first case the TPR with the noise injection circuit (NIC, a part of the multi-state radiometer [7]) was tested in several states of the circuit (Table 1). For each state the output noise temperature  $T_r$  was measured for twelve values of  $\Gamma_g$  and two values of  $T_g$  thus  $M = 24$ . It was assumed that the standard deviation of  $T_{\text{gexc}}$  was equal to 0.002 dB, the standard deviation of  $\Gamma_g$  was equal to 0.001, and the standard deviation of  $T_r$  was equal to 0.01 dB. For each state the mean square error (MSE) was calculated as the rms value of differences between the observed values of  $T_r$  and those predicted by the model. MSE may be treated as a measure of the quality of the fit. The values of  $s$  show whether the assumed levels of errors were underestimated ( $s > 1$ ) or overestimated ( $s < 1$ ). Since the values of  $s$  approach to one the error model may be considered accurate.

The obtained uncertainties of  $T_{\min}$  and  $T_N$  significantly depend on the state of the NIC, from 0.5% for the states In and ma up to 9% for the state Bs. This effect results from the distribution of  $\Gamma_g$  on the Smith chart. For the first two states the value of  $\Gamma_{\text{opt}}$  approximately lay in the center of the  $\Gamma_g$  constellation so the system of Eqs. (5) was well-conditioned. Since the same constellation was used for all states of NIC, for some of them the value of  $\Gamma_{\text{opt}}$  departed far from the center of the  $\Gamma_g$  constellation which may have deteriorated the conditioning.

In the next example two independent calibrations of TPR with NIC are compared (Fig. 2). The confidence intervals of both sets of parameters are in a good agreement. Similar results were obtained for other frequencies.

Moreover, the uncertainties obtained using this method agreed with ones calculated using perturbation method.



**Fig. 2.** Comparison of two calibrations of the TPR with NIC: (a)  $T_{\min}$  and (b)  $T_N$ ,  $f = 1$  GHz.

As the final case the TPR with an FET at the input was measured at several operating points (Table 2). For each bias point the output noise temperature  $T_r$  was measured for twelve cold states of the generator with  $\Gamma_g < 0.85$  and two hot ones with  $\Gamma_g \sim 0$ . The values of  $\text{var } T_{\text{gexc}}$ ,  $\text{var } \Gamma_g$ , and  $\text{var } T_r$  were like those in the first case but they seem to be underestimated ( $s$  is up to 1.7). It is also worth noting that  $\text{unc } T_{\min}$  and  $\text{unc } T_N$  get smaller when  $I_D$  increases. This can be explained as an improvement of conditioning for  $\Gamma_{\text{opt}}$  approaching the center of the  $\Gamma_g$  constellation.

## 5. Conclusions

The presented method for two-port noise parameter identification allows one to estimate the uncertainties of the parameters and their correlations. A simplified additive error model, which the method is based on, very well agrees with errors observed in experiments. The procedures for

Table 1  
The results of calibration of a TPR with a noise injection circuit,  $f = 2$  GHz

State	$T_{\min}$ [K]	unc $T_{\min}$ [K]	$T_N$ [K]	unc $T_N$ [K]	$ \Gamma_{\text{opt}} $ [mag]	$\angle\Gamma_{\text{opt}}$ [deg]	unc $\Gamma_{\text{opt}}$ [mag]	$ \Gamma_r $ [mag]	$\angle\Gamma_r$ [deg]	$g_r$ [dB]	MSE [dB]	$s$
In	482.5	2.0	731.6	4.0	0.1068	-139.0	0.0039	0.0895	163.7	-19.631	0.010	0.9
ma	488.0	5.2	8895.5	44.3	0.1112	-113.7	0.0021	0.0845	161.6	-19.687	0.015	1.1
Ao	2463.1	68.4	6044.8	131.1	0.7223	40.6	0.0044	0.0792	152.9	-19.719	0.008	0.7
As	1281.1	76.9	3760.8	150.2	0.7133	-150.2	0.0081	0.0850	160.7	-19.647	0.012	1.0
Bo	1404.0	55.4	4025.8	104.1	0.6543	142.4	0.0069	0.0881	160.3	-19.646	0.014	1.3
Bs	1502.9	137.9	7505.2	323.6	0.6692	-64.7	0.0095	0.0820	161.0	-19.721	0.015	1.3
Co	1308.9	58.2	5148.7	114.9	0.6000	97.9	0.0065	0.0921	157.6	-19.665	0.014	1.3
Cs	927.1	21.1	5965.8	96.8	0.6166	-108.5	0.0039	0.0844	161.2	-19.676	0.013	1.0
Do	870.6	26.1	5765.6	95.4	0.6037	-117.6	0.0044	0.0850	160.9	-19.671	0.013	1.1
Ds	997.7	34.6	7702.7	66.4	0.5290	38.2	0.0031	0.0863	156.4	-19.707	0.011	0.8

Table 2  
The results of calibration of TPR with a FET (MGF1412),  $f = 2$  GHz,  $V_{DS} = 3$  V

$I_D$ [mA]	$T_{\min}$ [K]	unc $T_{\min}$ [K]	$T_N$ [K]	unc $T_N$ [K]	$ \Gamma_{\text{opt}} $ [mag]	$\angle\Gamma_{\text{opt}}$ [deg]	unc $\Gamma_{\text{opt}}$ [mag]	$ \Gamma_r $ [mag]	$\angle\Gamma_r$ [deg]	$g_r$ [dB]	MSE [dB]	$s$
2	79.6	19.4	143.5	34.2	0.8746	-163.3	0.0223	0.9692	157.8	-15.068	0.007	1.1
5	50.9	15.2	95.3	31.0	0.8458	-162.0	0.0357	0.9622	153.2	-11.258	0.012	1.6
10	45.9	11.8	83.1	25.5	0.8181	-161.1	0.0392	0.9499	149.5	-9.070	0.012	1.7
20	45.4	8.2	83.3	18.3	0.7967	-158.8	0.0314	0.9424	146.0	-7.520	0.010	1.4
50	65.8	7.0	121.5	16.0	0.7655	-152.8	0.0223	0.9311	141.4	-6.207	0.007	1.0

calculating the uncertainties are tightly connected with the general least squares algorithm of noise parameters estimation. The numerical overheads related to the presented method are much smaller than in the perturbation method. The uncertainties of the two-port noise parameters calculated using the presented method may be used as a quick validation of the quality of a chosen  $\Gamma_g$  constellation. If implemented in the automatic measurement systems it may be utilised as a criterion for on-line optimization of the design of experiment, which may be extended or shrunk as needed for a given measurement accuracy. It may also facilitate finding out the most critical elements to be improved in order to minimise the sensitivity of the system to errors.

## Acknowledgment

The work was supported by the Maria Curie Skłodowska Curie Joint Found II under grant MEN/NIST-96-281 and by KBN under grant 1370/T10/99/16.

## References

- [1] J. W. Archer and R. A. Batchelor, "Fully automated on-wafer noise characterization of GaAs MESFET's and HEMT's", *IEEE Trans. Microw. Theory Techn.*, vol. 40, no. 2, pp. 209–216, 1992.
- [2] R. Benelbar, B. Huyart, and R. G. Bosio, "Microwave noise characterization of two-port devices using an uncalibrated tuner", *IEEE Trans. Microw. Theory Techn.*, vol. 44, no. 10, pp. 1725–1728, 1996.
- [3] S. Van den Bosch and L. Martens, "Improved impedance-pattern generation for automatic noise-parameter determination", *IEEE Trans. Microw. Theory Techn.*, vol. 46, no. 11, pp. 1673–1678, 1998.
- [4] A. C. Davidson, B. W. Leake, and E. W. Strid, "Accuracy improvements in microwave measurement of noise parameters", *IEEE Trans. Microw. Theory Techn.*, vol. MTT-37, pp. 1973–1977, 1989.
- [5] G. Mamola and M. Sannino, "Errors in measurement of microwave transistor noise parameters", *Alta Freq.*, vol. XLII, no. 10, pp. 551–556, 1973.
- [6] M. Schmidt-Szałowski, "Experimental verification of measurement uncertainties of multi-state radiometer system", in *Conf. MIKON'96*, Warsaw, Poland, 1996, pp. 331–335.
- [7] M. Schmidt-Szałowski and W. Wiatr, "An improved method for simultaneous small-signal and noise characterization of two-ports using multi-state radiometer", in *Conf. EuMC'99*, Muenchen, Germany, 1999, pp. 61–64.
- [8] B. N. Taylor and C. E. Kuyatt, "Guidelines for evaluating and expressing the uncertainties of NIST measurement results", *NIST Techn. Note*, no. 1297, 1994.
- [9] E. F. Vonesh and V. M. Chinchilli, *Linear and Nonlinear Models for the Analysis of Repeated Measurements*. New York, Basel, Hong Kong: Marcel Dekker, Inc. 1997.

**Marek Schmidt-Szałowski** received the M.Sc. and Ph.D. degrees in electronic engineering from Warsaw University of Technology (Poland) in 1995 and 2000, respectively. His theses concerned novel measurement techniques for small-signal and noise characterization of microwave devices. He is a co-inventor of the multistate radiometer. In 2000, he joined Philips' Business Line RF Modules (Nijmegen, The Netherlands). His responsibilities include characterization and modeling of RF semiconductor devices. He is presently engaged in characterization of high-frequency low-noise SiGe BJT's.

Institute of Electronic Systems  
Warsaw University of Technology  
Nowowiejska st 15/19  
00-665 Warsaw, Poland

**Wojciech Wiatr** received the M.Sc., and the Ph.D. degrees in electronic engineering from Warsaw University of Technology, Warsaw, Poland in 1970 and 1980, respectively. In 1971, he joined the Institute of Electron Technology CEMI in Warsaw, affiliated to national semiconductor industry. Since 1972, he has been with the Institute of Electronic

Systems at the Warsaw University of Technology, presently as an Assistant Professor. His main scientific interests are in the field of precision microwave measurements. He developed new techniques and instrumentation for broadband scattering and noise parameter measurements of microwave transistors and MMICs. Recently, he invented the RF multistate total power radiometer realizing simultaneous noise and vector analysis of microwave networks with natural noise excitation. He was engaged in realization of many research projects. In noise metrology, he collaborated with the National Radio Astronomy Observatory in Charlottesville, VA, USA and the Ferdinand-Braun-Institut für Höchstfrequenztechnik in Berlin. For five years, he has been an Affiliate Researcher of the National Institute of Standards and Technology in Boulder, CO, USA. Dr. Wojciech Wiatr published 1 book, over 70 papers and holds two patents.

e-mail: Wiatr@ise.pw.edu.pl  
Institute of Electronic Systems  
Warsaw University of Technology  
Nowowiejska st 15/19  
00-665 Warsaw, Poland